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Separation Speed of Undamped Metal Bellows Contacting Mechanical Face Seals©

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Contacting mechanical seals are load unbalanced. The unbalanced load (the difference between closing and opening forces) causes the seal faces to be in mechanical contact suppressing fluid pressure generation in the sealing dam. The two major concerns with contacting mechanical face seals are the wear rate of the faces and whether these faces will separate while in operation. These two modes of failure are directly related to the amount of initial compression (i.e., preset) in the bellows and the hydraulic loading. A new criterion for separation is presented based on a pragmatic model of the mechanics of contact. The separation speed derived herein is less conservative than that found more than two decades ago. The optimal speed which minimizes wear is determined to occur at the resonance frequencies of the system, thus providing guidelines for improved designs.

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INTRODUCTION

Contacting mechanical seals are load unbalanced where the closing force on the flexibly mounted element (due to hydraulic and spring loading) exceeds the opening force (due to presumed fluid film effects). The unbalanced load results in face contact. Applications such as automotive water pump seals, stuffing box naval seals, light hydrocarbon seals (being heavily spring loaded to prevent fluid flashing), and unpressurized gas seals are just a few examples. In a recent failure analysis of a hot black liquor seal, micro and macro cracks were found in the carbon ring along with severe surface damage. It was concluded that vigorous shaft vibration caused material fatigue in approximately two weeks of operation. This case prompted the current analysis.

A contacting mechanical face seal whose stationary primary seal ring is supported by a metal bellows is shown in Fig. 1. The seal ring, stator, and mating ring, rotor, are maintained in contact resulting from a preset in the bellows. Usually the two major causes of failure in contacting face seals are wear of the faces and separation of the faces during

NOMENCLATURE

d = radial location of the center of pressure
F = force at the point of contact during assembly
F_c = force which causes contact onset
F_p = a component of the total force caused by a non-uniform pressure distribution
F_{total} = total load on the stator resulting from the interface
F_Δ = additional preset force
F_ς = compression force in axial mode
I = stator transverse moment of inertia
K = axial support stiffness
k = angular support stiffness
M = moment resulting from the force of contact during assembly
m = stator mass
M_c = moment resulting from the force which causes contact onset
M_p = moment on the stator while in operation, resulting from the interface
= total moment on the stator resulting from the interface
R = radial location at which the stator and rotor make contact
r_s = radial location of flexible support
t = time
Z = stator axial degree of freedom
Z_a, Z_b = stator axial displacements at pitch points a and b, respectively
Z_c = minimum axial preset required to cause contact onset
γ_r = rotor misalignment (runout)
γ_s = stator tilt (nutation)
ΔF = a component of the total force caused by a uniform pressure distribution
ΔR = sealing dam (interface) width
ΔZ = flexible support axial preset
ς = axial pulsation of the shaft
ς_o = amplitude of axial pulsation of the shaft
ψ = precession
ω = shaft angular velocity
ω_n = natural frequency
ω_{sep} = shaft angular velocity to cause separation onset
= frequency of axial pulsation of the shaft

operation. Excessive friction, resulting from a high bellows preset, will wear the faces rapidly, while low bellows preset may be insufficient to maintain contact, i.e., prevent separation. In either case, sealing capability is damaged, and seal failure is imminent. Hence, it is desirable to determine the right level of bellows preset for a given set of operating conditions and system parameters. These include rotor runout (caused by manufacturing and assembly tolerances), stator inertia, speed, and metal bellows stiffness.

Only four known works have addressed the above stated problem. Hart and Zorowski (*1*) determined the onset of separation assuming the bellows to be represented by a series of distributed springs and dampers supporting a rigid seal carrier. A major assumption in their work was that the contact produces a linear compressive force distribution across the faces. Separation would occur when the minimum compressive force reduces to zero. In a subsequent work, Hart and Zorowski (*2*), included face friction and bellows torsion. Zorowski and Hill (*3*) extended the above analyses by relaxing the surface rigidity to include interface axial and torsional elasticity. This was done by representing the interface with only four springs, where separation would occur when the internal compression force in one of the springs would become zero. The equations of motion were then solved on an analog computer to simulate the dynamic behavior of the seal, and maps describing separation were provided. In a recent work, Lipschitz (*4*) has highlighted important factors such as hydraulic loading and eccentricity.

The intent of the present work is to examine once again the basic problem where the faces are perfectly rigid, the rotor is misaligned, the undamped bellows is preset, and the shaft is rotating at a constant speed. The separation speed will be found analytically based on a pragmatic, but quite realistic, model of the mechanics of contact. It will also be shown that there are situations in which a certain portion of the bellows can be in tension yet still maintain full contact of the faces. The criterion for separation suggested here differs from those offered in previous works and, therefore, leads to a different separation speed.

(It should be noted that once separation has occurred, and a fluid film has permanently been generated, the seal would be classified as noncontacting, and one may wish to resort to an alternate analysis, e.g., Green and Etsion (*5*). It is, therefore, assumed herein that full contact prevails, and if a film does exist, say between surfaces asperities, its load carrying capacity is insignificant compared to other existing forces.)

ANALYSIS

Axial Mode

Experimental evidence of pulsation along the axis of the shaft has never been reported to be significant in seal performance. For the reason to be determined herein, however, axial pulsation should not be ignored when considering the dynamic behavior of seals in general and contacting seals in particular.

Figure 2(a) shows schematically the axial pulsation mode where K is the axial stiffness of the metal bellows, m is the mass of the stator, and ΔZ is the bellows preset. The forcing function in the axial mode results from ς_o and ω_ς, which are the amplitude and frequency of the axial pulsation of the shaft, respectively. Hence,

$$\varsigma = \varsigma_o \sin \omega_\varsigma t \qquad [1]$$

Figure 2(b) presents a free body diagram, where F_ς is the compression force, and Z is the axial degree of freedom. The equation of motion in the axial mode is simply

$$m\ddot{Z} = -K(Z + \Delta Z) + F_\varsigma \qquad [2]$$

As long as contact exists, $Z = \varsigma$. Hence, Eq. [1] results in

$$\ddot{Z} = -\varsigma_o \omega_\varsigma^2 \sin \omega_\varsigma t$$

and the compression force is solved from Eq. [2], yielding

$$F_\varsigma = \varsigma_o \sin \omega_\varsigma t\,(K - m\omega_\varsigma^2) + K\Delta Z \qquad [3]$$

To maintain contact at all times the compression force must never vanish, i.e., $F_\varsigma > 0$. Since $\sin \omega_\varsigma t$ is bounded by $\pm$ 1, the condition to assure contact becomes

$$\Delta Z > \varsigma_o \left| \left(\frac{\omega_\varsigma}{\omega_n}\right)^2 - 1 \right| \quad @ \; \omega_n^2 = \frac{K}{m} \qquad [4]$$

where ω_n is the natural frequency of the system in the axial mode. At very low forcing frequencies a quite predictable result is obtained that the preset should slightly exceed the axial pulsation amplitude. However, the required preset decreases as the forcing frequency increases as long as the forcing frequency is kept below the natural frequency. The

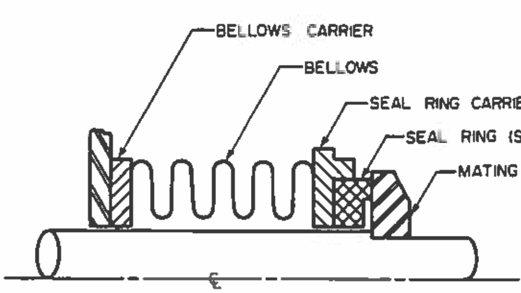

Fig. 1—Schematics of a metal bellows contacting mechanical face seal.

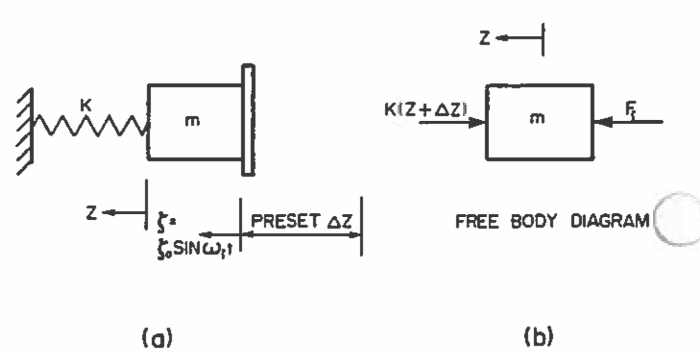

Fig. 2—Lumped parameter model for the axial pulsation mode.

optimal case is when the forcing and natural frequencies are equal; that is $\omega_s = \omega_n$. In this case the necessary preset to prevent separation according to Eq. [4] becomes zero. Thus, face wear, which is directly effected by face loading caused by spring preset, is minimal. This result is of interest since the optimal performance in the axial mode occurs at resonance. As ω_s increases above $2\omega_n$ the required ΔZ increases, indicating that forcing frequencies much higher than the system natural frequency should be avoided.

Angular Mode

To obtain the separation speed in the angular mode it is necessary to understand the mechanics of contact. Figure 3(a) shows a misaligned rotor prior to assembly, where γ_r represents the rotor misalignment. First contact is made at a point at the bottom of the two faces, where F is the contacting force at a point which causes the stator to deflect axially by an amount Z and rotate through an angle γ_s, see Fig. 3(b). A free body diagram for this intermediate situation is shown in Fig. 3(c) where the force, F, has been moved to the stator center of mass, and a couple, $M = FR$, has been added. In the work by Green and Etsion (5) (Appendix 1) the angular stiffness coefficient, k, was determined from the corresponding axial stiffness coefficient, K, to be

$$k = \frac{1}{2} K r_s^2 \qquad [5]$$

where r_s is the radial location of the elastic support, i.e., the metal bellows radius in this case. Therefore, with the aid of Fig. 3(c) the force and moment are written as

$$F = KZ \qquad [6a]$$

$$M = k\gamma_s = FR \qquad [6b]$$

where R is the radial location at which the stator makes contact with the rotor. For conciseness it will be assumed that $R = r_s$. A requirement commonly imposed upon contacting seals is that contact must exist under all operation conditions including rest. Figure 4 illustrates the onset of contact over the entire surface of both the stator and the rotor, but the force which causes this contact, F_c, remains concentrated at point b. In this situation the stator nutation completely assumes the rotor misalignment angle such that $\gamma_s = \gamma_r$. Again the contacting force is moved to the stator mass center yielding the couple

$$M_c = F_c \cdot R \qquad [7]$$

Using Eqs. [6] for the onset of contact results in

$$F_c = KZ_c \qquad [8a]$$

$$M_c = k\gamma_r \qquad [8b]$$

Substituting Eqs. [8] in Eq. [7] yields

$$\frac{1}{2} KR^2\gamma_r = KZ_c R \qquad [8]$$

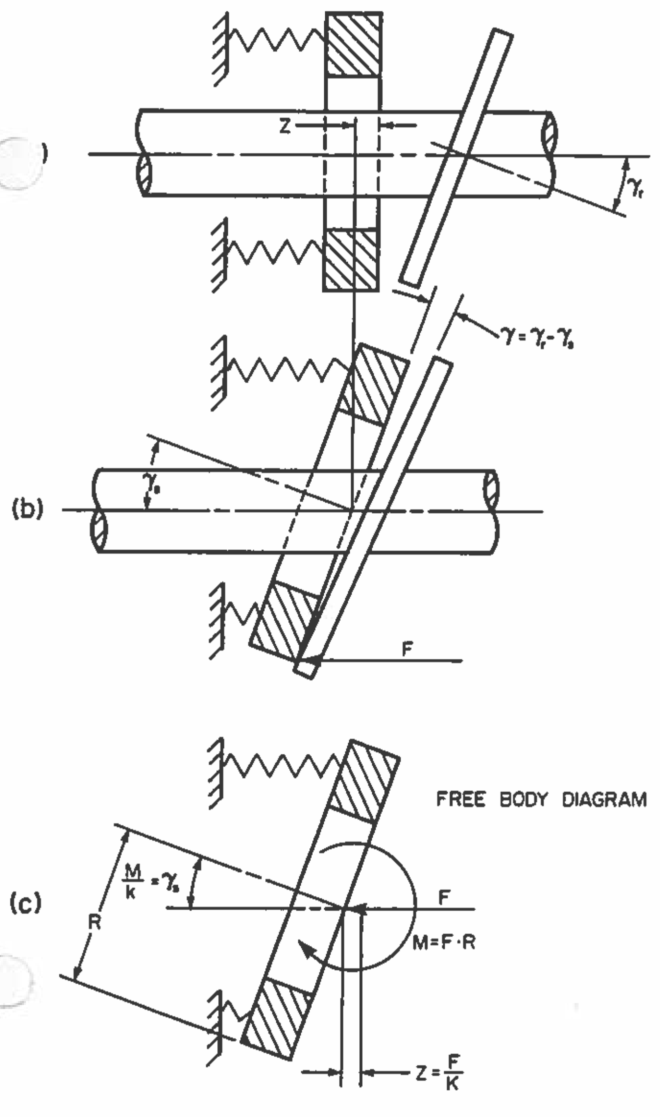

Fig. 3—Stages of assembly.

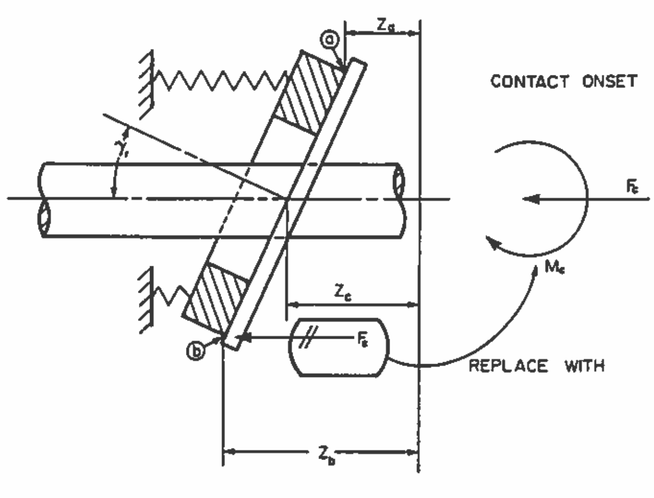

Fig. 4—Contact onset.

where k has been replaced by its equivalent as given by Eq. [5]. Equation [8] simplifies to

$$Z_r = \frac{1}{2} R\gamma_r \qquad [9]$$

where Z_r is the axial preset of the metal bellows required to cause the onset of full contact. By examining Fig. 4 the deflection can be obtained at every point on the circumference of the two faces. Designating Z_a and Z_b as the extreme points of minimum and maximum deflection, respectively, one gets

$$Z_a = Z_c - R\gamma_r = -\frac{1}{2} R\gamma_r \qquad [10a]$$

$$Z_b = Z_c + R\gamma_r = \frac{3}{2} R\gamma_r \qquad [10b]$$

Note that the fact that the rotor misalignment is very small has been utilized. This results in a very small stator nutation. Figure 5 is a graphical representation of the location of points Z_a, Z_b, and Z_c. The metal bellows is compressed along three quarters of its circumference, but it is stretched along one quarter of the circumference indicating a tensile force in this region. Z_b represents maximum compression, while Z_a represents maximum stretch. (From a fatigue design standpoint it is recommended to eliminate the tensile stress in the bellows by adding 1/2 $R\gamma_r$ to the axial preset, Z_c.) The equations of motion must be investigated in order to obtain the separation speed. The dynamic moments in these equations have been derived elsewhere (*6*). Hence,

$$I\left(\ddot{\gamma}_s - \dot{\psi}^2\gamma_s\right) = M_x \qquad [11a]$$

$$I\left(\ddot{\psi}\gamma_s + 2\dot{\gamma}_s\dot{\psi}\right) = M_y \qquad [11b]$$

where M_x and M_y are the applied moments that act upon the stator, x designating the axis about which the stator is nutated and y designating the axis perpendicular to x in the stator plane. Both precess at a rate $\dot{\psi}$. The transverse moment of inertia is designated by I, and the shaft angular velocity by ω. As long as contact is maintained, the following kinematic conditions prevail:

$$\dot{\psi} = \omega = \text{const} \qquad [12a]$$

$$\gamma_s = \gamma_r = \text{const} \qquad [12b]$$

Hence,

$$\ddot{\gamma}_s = \dot{\gamma}_s = 0\ ;\ \ddot{\psi} = 0 \qquad [12c]$$

By substituting conditions [12] into Eqs. [11], only one equation about x is left

$$-I\omega^2\gamma_r = -k\gamma_r + M_p \qquad [13]$$

The equation of motion in the y direction is identically zero because for an undamped support system the applied moment about y does not exist. The left hand side of Eq. [13] results from inertia effects. The right hand side consists of the bellows restoring moment, $-k\gamma_r$, and the moment, M_p, which originates at the interface between stator and rotor. M_p is positive about x.

Equation [13] can be rearranged by applying D'Alembert's principle, and becomes

$$M_p + I\omega^2\gamma_r - k\gamma_r = 0$$

From this equation it is evident that the inertia term, $I\omega^2\gamma_r$, has the nature of a nonrestoring moment, tending to increase stator nutation. (This tendency is not necessarily detrimental, as will be seen later.)

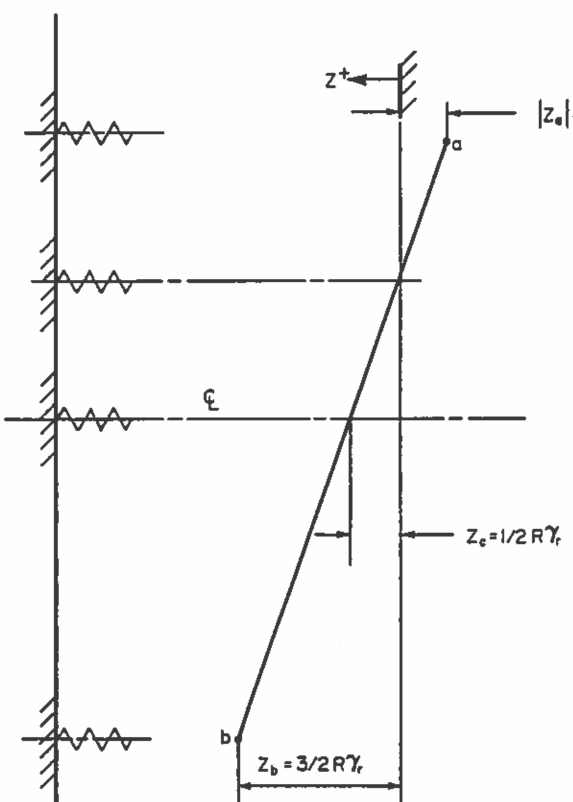

Fig. 5—Axial displacement of various points on the circumference at contact onset

Separation Onset

Separation onset will be considered in two stages: without additional preset, and with additional preset.

Stage I: No Additional Preset

It is assumed that the stator and rotor are rigid and that the only preset in the bellows is Z_c as given by Eq. [9]. This value of Z_c originates from the force F_c, which causes contact onset. At this position the shaft is locked and is restrained from motion in the axial direction (as is commonly done in practice). Solving for M_p from Eq. [13] yields

$$M_p = \gamma_r \left(k - I\omega^2\right) \quad [14]$$

At rest, $\omega = 0$, one returns to the condition of Fig. 5 and Eqs. [7] and [8] such that $M_p = M_c$ where all the contacting force is concentrated at a point. This situation is shown in Fig. 6(a). As the speed increases, M_p decreases according to Eq. [14]. Hence, the contacting force ceases to be concentrated at a point; rather, the load is carried by the asperities in the interface between stator and rotor. The distribution of the contacting force depends on the topography of the interface and the material properties of the stator and rotor. This is a statically indeterminate problem that is not easily solved. However, in order to determine the speed at which separation will occur, it is assumed that the contact establishes a pressure in the interface which varies only circumferentially; i.e., $P = P(\theta)$. Figure 6(b) shows a side view of such a pressure profile applied to the interface, which is tilted about the rotating axis of nutation, x, such that the interface precesses with the stator and the rotor. The overall force that acts on the stator is given by

$$F_{total} = 2R\Delta R \int_0^{\pi} P(\theta)\, d\theta \quad [15]$$

and the total moment, is given by

$$M_{total} = 2R^2\, \Delta R \int_0^{\pi} P(\theta) \cos\theta\, d\theta \quad [16]$$

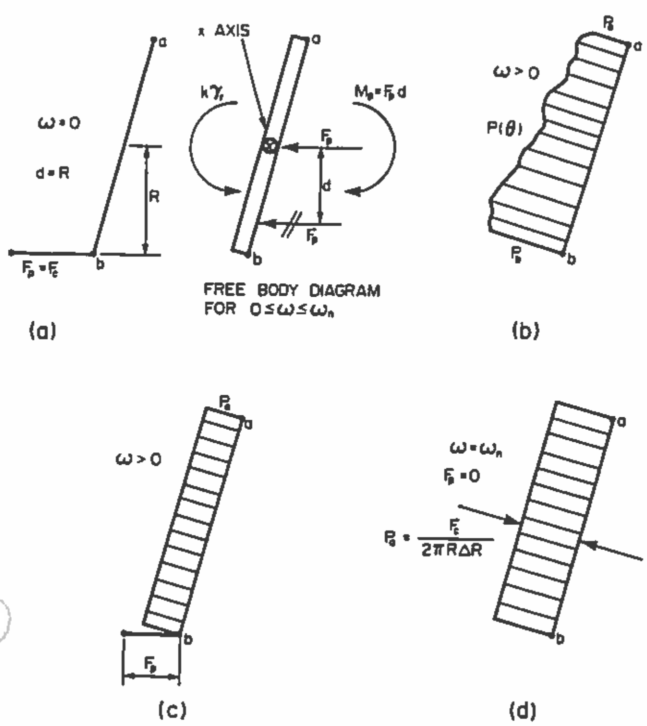

Fig. 6—Pressure distribution, concentrated forces, and center of pres-

where ΔR represents the interface contacting width.

It is quite obvious that as long as there is contact over the entire interface, the total force, F_{total}, must equal the onset contacting force, F_c. ($F_{total} > F_c$, is the condition of face separation.) The total force is divided into two component forces: ΔF which represents a force due to a uniform pressure distribution, and F_p which represents a force caused by the non-uniformity of the contact pressure. For both, the following applies:

$$\Delta F = 2\pi R\, \Delta R\, P_a \quad [17]$$

and

$$F_p = 2R\, \Delta R \int_0^{\pi} [P(\theta) - P_a]\, d\theta \quad [18]$$

such that

$$F_{total} = F_c = \Delta F + F_p \quad [19a]$$

where P_a is the uniform pressure and equals the minimum pressure at point 'a' in Fig. 6(b). It is obvious to see that the total moment in Eq. [16] is unaffected by P_a, where

$$M_{total} = M_p = 2R^2\, \Delta R \int_0^{\pi} [P(\theta) - P_a] \cos\theta\, d\theta \quad [19b]$$

It is now possible to define a radial location, d, of a center of pressure such that $d = M_p/F_p$, as if the entire force F_p is concentrated at that point. For convenience, it will be written as

$$F_p = M_p/\mathrm{d} \quad [20]$$

Substituting Eq. [14] into [20], yields

$$F_p = \frac{\gamma_r}{d}(k - I\omega^2) \quad [21]$$

It is unnecessary to know the exact function $P(\theta)$, in order to solve the problem. Generally, however, the pressures profile should have a larger magnitude in the lower portion of Fig. 6(b) than that in the upper portion in order to produce a positive moment, M_p, about x. (For the reason that separation onset will transpire when the entire load on the stator is concentrated at a point, the problem, if one wishes, can be examined alternately by assuming perfectly rigid stator and rotor with smooth flat faces. Then there is a uniform pressure over the entire circumference, and a concentrated force, F_p, at the bottom point, 'b', as shown in Fig. 6(c). In this case $d = R$ at all times. It is important to note, however, that the analysis is not limited to this idealization.)

Since F_p is associated with a concentrated force, which increases the wear of the faces, it is desirable to minimize F_p or to eliminate it altogether. This will occur when $k - I\omega^2 = 0$ as evident from Eq. [21], yielding $F_p = 0$. In this case M_p is also zero as evident from Eq. [14]. Hence, be uniform so that $P(\theta) = P_a = P_b =$ const., as depicted in Fig. 6(d). If the transverse moment of inertia is assumed to be $I = mR^2/2$ (which is a close approximation for many practical seals) the best performance exists when $k - I\omega^2 = 0$, or

$$\omega^2 = \frac{k}{I} = \frac{\frac{1}{2}KR^2}{\frac{1}{2}mR^2} = \frac{K}{m} = \omega_n^2 \qquad [22]$$

where k has been replaced by its equivalent from Eq. [5] (recall that $r_3 \equiv R$). This result is quite interesting since operation at resonance frequency again appears to produce the best dynamic behavior, as every point in the interface participates equally in carrying the closing force, F_c.

When $\omega > \omega_n$ one finds that $k - K\omega^2 < 0$, and, as evident from Eq. [14], $M_p < 0$. The interpretation of a negative moment is actually a change in its direction. That is, the interface must induce a moment in the negative x direction. Defining $\overline{M}_p = -M_p$, yields

$$\overline{M}_p = \gamma_r\,(I\omega^2 - k) \qquad [23]$$

and as the speed, ω, increases, $\overline{M}_p$ increases. This causes the pressure profile, $P(\theta)$, to have its higher values in the upper part of the stator as depicted in Fig. 7(a), or for the idealized problem, see Fig. 7(b). Hence, the concentrated force F_p has moved from the lower part to the upper part. This is not surprising, however, since the inertia effect, as indicated earlier, has the role of a non-restoring moment which tends to increase stator nutation. The interface resists this tendency by applying $\overline{M}_p$. The only way in which $\overline{M}_p$ can increase with speed, while full contact is maintained, is for F_p and d to increase with speed, since $\overline{M}_p = F_p d$. (Still, $F_{total} = F_c$ and the stator mass center has not moved.) Thus, the center of pressure is moving towards point 'a' on the circumference. The limiting case is when the speed is high enough to concentrate the entire force at point 'a', as shown in Fig. 7(c), which will result in the onset of separation. In this case $d = R$, $F_p = F_c$ (any force F_p bigger than F_c will move the center of mass further along axis Z to cause opening between stator and rotor), and $\omega = \omega_{sep}$. Using Eq. [23] results in

$$\gamma_r\,(I\omega^2_{sep} - k) = F_c \cdot R \qquad [24]$$

Using Eqs. [7] and [8b] to replace F_c with $k\gamma_r/R$, yields

$$\gamma_r\,(I\omega^2_{sep} - k) = k\gamma_r \qquad [25]$$

Solving for the separation speed gives

$$\omega^2_{sep} = \frac{2k}{I} = 2\omega_n^2 \qquad [26]$$

or

$$\omega_{sep} = \sqrt{2}\,\omega_n \qquad [27]$$

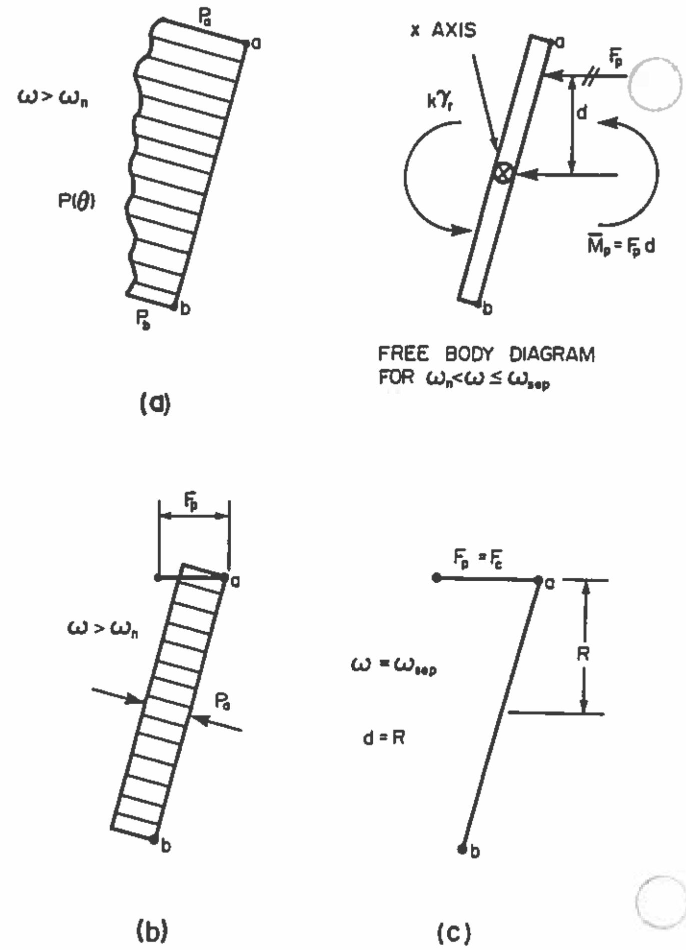

Fig. 7—Pressure distribution, concentrated forces, and center of pressure for the speed range $\omega_n < \omega \le \omega_{sep}$.

State II: Additional Preset

It is useful to find the separation speed for the case when additional preset, ΔZ, is forced upon the bellows, as shown in Fig. 8. Such a preset might be useful to prevent separation in the axial mode as dictated by Eq. [4]. Additional preset might also be imposed to eliminate the tensile force in the bellows and to improve fatigue resistance, or it can simply be used as a safety factor to guard against separation.

Any deflection ΔZ beyond Z_c is caused by an additional force, F_Δ, where

$$F_{total} = F_c + F_\Delta = F_c + K\Delta Z \qquad [28]$$

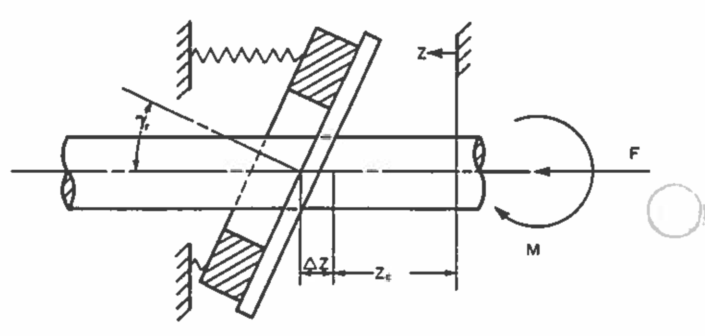

Fig. 8—Bellows preset beyond contact onset.

does not have an effect on M_p or $\overline{M}_p$, since ΔZ causes a uniform deflection of the bellows, thus increasing the magnitude of the uniform pressure, P_a. In other words, F_Δ is equally distributed around the interface to yield a zero net additional moment. Using the same arguments that led to Eq. [24], separation will occur only when the force that acts on the stator center of mass exceeds F_{total} (stored also in the bellows) as given by Eq. [28]. Hence,

$$\gamma_r\,(I\omega_{sep}^2 - k) = (F_c + F_\Delta)\,R \qquad [29]$$

Again replacing F_c with $k\gamma_r/R$, noting that $F_\Delta = K\Delta Z$, and using Eq. [5] $k = KR^2/2$, the separation speed is found to be

$$\begin{aligned}\omega_{sep}^2 &= 2\frac{k}{I}\left(1 + \frac{\Delta Z}{R\gamma_r}\right)\\ &= 2\omega_n^2\left(1 + \frac{\Delta Z}{R\gamma_r}\right)\end{aligned} \qquad [30]$$

or

$$\omega_{sep} = \omega_n\left[2\left(1 + \frac{\Delta Z}{R\gamma_r}\right)\right]^{1/2} \qquad [31]$$

Note that when $\Delta Z = 0$, Eq. [31] reduces to Eq. [27].

DISCUSSION AND CONCLUSIONS

Equation [31] provides a general criterion for determining the separation speed of undamped contacting mechanical face seals. It has been shown that in order to maintain full contact at rest, the flexible support, (e.g., metal bellows) must be preset at least an amount $Z_c = R\gamma_r/2$ where γ_r is the rotor runout. Since Z_c adversely affects wear it is recommended that γ_r be reduced as much as possible. For the same reason it is also recommended that any additional preset, ΔZ (or any hydraulic closing force that can be translated into ΔZ), be as small as possible, despite the fact that high values of ΔZ increase the separation speed value.

It has been found that inertia produces a non-restoring moment. This moment is advantageous as long as $\omega < \omega_n$, since it reduces localized wear due to high concentrated forces. It is most beneficial when $\omega = \omega_n$, since the contacting force is distributed evenly over the faces in this case. Furthermore, $\omega = \omega_n$ is always less than the separation speed, ω_{sep}, and therefore, it is desired to design the system so that the rotational natural frequency will be equal the shaft speed. Likewise, it has been shown that there is no need for an axial preset if the axial natural frequency equals the shaft axial pulsation frequency. At high shaft speeds where $\omega > \omega_n$, the non-restoring moment rotates the pitch point (about which the stator tends to increase its tilt) by 180° from bottom to top, and the wear conditions worsen with speed.

It is now of interest to compare the separation speed obtained in this analysis with that obtained by Hart and Zorowski (*1*) (see Appendix), given by

$$\omega_{sep} = \omega_n\left(1 \pm \frac{\Delta Z}{R\gamma_r}\right)^{1/2}$$

where the negative and positive signs correspond to low and high separation speeds, respectively. Hart and Zorowski explain that as long as $\Delta Z/R\gamma_r < 1$, the separation speed decreases with an increase in bellows compression. Moreover, $\Delta Z/R\gamma = 1$ results in zero separation speed to indicate imminent separation.

Obviously, the result obtained by Hart and Zorowski differs from that presented in Eq. [31]. The reason for the deviation originates with different separation criteria. Hart and Zorowski assume that "setting the contact force$(\overline{F}_i)_{min}$ (being the minimum compression force) equal to zero establishes the criterion for the onset of separation." The contacting mechanics model as presented here, e.g., Fig. 5 and 6(a) demonstrate that even if the minimum force is zero (actually it can be zero all over the interface except for one point) full contact can still be maintained. In order to initiate separation the present work suggests that the separating force that acts on the stator must be bigger than the one initially used to compress the stator and rotor together.

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APPENDIX

The separation speed criterion of metal bellows mechanical seals according to Hart and Zorowski (*1*) is given in their work by Eq. [21] as

$$\delta = \nu\,[(1 - \rho)^2 + \varepsilon^2\rho]^{1/2} + [(1 - \lambda\rho)^2 + \varepsilon^2\rho]^{1/2} \qquad [A1]$$

where

ν = dimensionless amplitude ratio, $A_o/r\theta_o$
ε = dimensionless damping constant
λ = dimensionless frequency ratio, $(\Omega_z/\Omega_\theta)^2$
ρ = dimensionless forcing frequency, $(\omega/\Omega_z)^2$
δ = dimensionless precompression parameter, $Z_o/r\theta_o$
A_o = displacement amplitude in Z direction
r = mean ring radius
θ_o = displacement amplitude about x
Ω_z = natural frequency of translational mode
Ω_θ = natural frequency of rotational mode

For the purpose of comparison it is assumed that shaft axial pulsation does not exist, $A_o = 0$, i.e., $\nu = 0$. To comply with the condition of the present investigation, damping does not exist by definition, i.e. $\varepsilon = 0$, and the natural frequencies in the translational and rotational modes equal each other, i.e., $\lambda = 1$. Hence, from Eq. [A1], one gets

$$\delta = \pm (1 - \rho) \qquad [A2]$$

Solving for ρ yields

$$\rho = 1 \pm \delta \qquad [A3]$$

Substituting the corresponding dimensional parameters gives

$$\omega_{srp} = \Omega_z \left(1 \pm \frac{Z_o}{r\theta_o}\right)^{1/2} \qquad [$$

Using nomenclature of this work to replace corresponding parameters in Eq. [A4] results finally in

$$\omega_{srp} = \omega_n \left(1 \pm \frac{\Delta Z}{R\gamma_r}\right)^{1/2} \qquad [A5]$$